nag_real_eigenvalues (f02afc)

1. Purpose

nag_real_eigenvalues (f02afc) calculates all the eigenvalues of a real unsymmetric matrix.

2. Specification

```
#include <nag.h>
#include <nagf02.h>

void nag_real_eigenvalues(Integer n, double a[], Integer tda,
     Complex r[], Integer iter[], NagError *fail)
```

3. Description

The matrix A is first balanced and then reduced to upper Hessenberg form using stabilised elementary similarity transformations. The eigenvalues are then found using the QR algorithm for real Hessenberg matrices.

4. Parameters

n
Input: n, the order of the matrix A.
Constraint: $\mathbf{n} \geq 1$.

a[n][tda]
Input: the n by n matrix A.
Output: the array is overwritten.

tda
Input: the second dimension of the array **a** as declared in the function from which nag_real_eigenvalues is called.
Constraint: $\mathbf{tda} \geq \mathbf{n}$.

r[n]
Output: the eigenvalues.

iter[n]
Output: **iter**$[i-1]$ contains the number of iterations used to find the ith eigenvalue. If **iter**$[i-1]$ is negative, the ith eigenvalue is the second of a pair found simultaneously.
Note: the eigenvalues are found in reverse order, starting with the nth.

fail
The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_TOO_MANY_ITERATIONS
More than ⟨*value*⟩ iterations are required to isolate all the eigenvalues.

NE_INT_ARG_LT
On entry, **n** must not be less than 1: $\mathbf{n} = \langle value \rangle$.

NE_2_INT_ARG_LT
On entry, $\mathbf{tda} = \langle value \rangle$ while $\mathbf{n}$ = value. These parameters must satisfy $\mathbf{tda} \geq \mathbf{n}$.

NE_ALLOC_FAIL
Memory allocation failed.

6. Further Comments

The time taken by the function is approximately proportional to n^3.

6.1. Accuracy

The accuracy of the results depends on the original matrix and the multiplicity of the roots. For a detailed error analysis see Wilkinson and Reinsch (1971) pp 352 and 367.

6.2. References

Wilkinson J H and Reinsch C (1971) *Handbook for Automatic Computation (Vol II, Linear Algebra)* Springer-Verlag pp 339–371.

7. See Also

None.

8. Example

To calculate all the eigenvalues of the real matrix

$$\begin{pmatrix} 1.5 & 0.1 & 4.5 & -1.5 \\ -22.5 & 3.5 & 12.5 & -2.5 \\ -2.5 & 0.3 & 4.5 & -2.5 \\ -2.5 & 0.1 & 4.5 & 2.5 \end{pmatrix}.$$

8.1. Program Text

```
/* nag_real_eigenvalues(f02afc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagf02.h>

#define NMAX 4
#define TDA NMAX
#define COMPLEX(A) A.re, A.im

main()
{
  Integer  i, j, n;
  double a[NMAX][TDA];
  Complex r[NMAX];
  Integer iter[NMAX];

  Vprintf("f02afc Example Program Results\n");
  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vscanf("%ld",&n);
  if (n<1 || n>NMAX)
    {
      Vfprintf(stderr, "N is out of range: N = %5ld\n", n);
      exit(EXIT_FAILURE);
    }
  for (i=0; i<n; i++)
    for (j=0; j<n; j++)
      Vscanf("%lf", &a[i][j]);
  f02afc(n, (double *)a, (Integer)TDA, r, iter, NAGERR_DEFAULT);
  Vprintf("Eigenvalues\n");
  for (i=0; i<n; i++)
    Vprintf("( %7.3f  , %7.3f ) \n", COMPLEX(r[i]));
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
f02afc Example Program Data
  4
   1.5   0.1   4.5  -1.5
 -22.5   3.5  12.5  -2.5
  -2.5   0.3   4.5  -2.5
  -2.5   0.1   4.5   2.5
```

8.3. Program Results

```
f02afc Example Program Results
Eigenvalues
(   3.000  ,   4.000 )
(   3.000  ,  -4.000 )
(   4.000  ,   0.000 )
(   2.000  ,   0.000 )
```